Exercise 90

Suppose that the price per unit in dollars of a cell phone production is modeled by $p = \$45 - 0.0125x$, where x is in thousands of phones produced, and the revenue represented by thousands of dollars is $R = x \cdot p$. Find the production level that will maximize revenue.

Solution

Substitute the formula for p into the one for R and complete the square to write the quadratic function in vertex form.

$$\begin{aligned}
R &= xp \\
&= x(45 - 0.0125x) \\
&= 45x - 0.0125x^2 \\
&= -0.0125(x^2 - 3600x) \\
&= -0.0125[(x^2 - 3600x + 1800^2) - 1800^2] \\
&= -0.0125[(x - 1800)^2 - 1800^2] \\
&= -0.0125(x - 1800)^2 + 0.0125(1800^2) \\
&= -0.0125(x - 1800)^2 + 40\,500
\end{aligned}$$

Therefore, the maximum revenue is $R = 40\,500$ (\$40,500,000), which occurs when $x = 1800$ (1,800,000 phones) and $p = 45 - 0.0125(1800) = 22.5$.